Making Decisions Under Risk – Prospect Theory Approach

Introduction

Decision making is an inherent part of human existence. At times it can even determine survival as natural selection is based on vigilance and approach-avoidance reactions. Decision making is also one of the cardinal domains in economics which concerns economic processes and rights as much as regularities embedded within these processes. Decision making aims at showing how people, namely decision-makers, who act in diverse socio-economic conditions use and apply their human or natural resources in various situations. It examines what these people are driven by, how they make decisions and, last but not least, if they are effective. We need to bear in mind, however, that economics very often deals with simplified models that are not necessarily in line with real-life phenomena. Hence, it is not always possible to explain thoroughly how the choices are made.

There is an alternative to traditional economics which creates non-contextual models of economics based on a set of primal, self-evident rules that need no empirical verification at all - behavioral economics. It exploits the so called realistic approach which is a compromise between one applying inductive reasoning and analytical approach. The adjective behavioral derives from the fact that realism is obtained by modeling based on a reality set (i.e. the observation of behavior that is real) Basically it means that theories in behavioral economics are contextual, in other words they are rooted in a given particular social reality with its culture, institutions or systems of values.

Interdisciplinarity is an essential feature of behavioral economics as, while observing the consumers (their choice etc.), it builds on sociological and psychological research and, at the same time, it requires integration of knowledge derived from social science. Thus, the important issue of

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3. Any individual (a consumer, manager, etc) or institution (a commune, company, etc) that makes decisions may be decision-makers; T. Tyszka, Decyzje. Perspektywa psychologiczna i ekonomiczna, Wydawnictwo Naukowe SCHOLAR, Warszawa 2010, p. 48.
4. R. Milewski, E. Kwiatkowski, Podstawy ..., op. cit., p. 3.
decision-making under risk will be studied in the hereby presented paper. It will be discussed on the basis of the prospect theory that contributes significantly to the “modern” approach to economics.

1. Decision making theories

The modes of making decisions under risk and uncertainty have been discussed for centuries. They accelerated in the 17th century when a French nobleman C. de Mere posed a question to Pascal and provoked him to develop a strategy of a gambling game leading to little gain in a longer perspective of time. In his response to the question Pascal formed expected value principle. It is the average distribution of results and according to the principle the value of a future gain should be directly proportional to the chance of getting it. Expectations are the predictions of future events based on information, so by and large they are the same as predictions provided by an adequate economic theory. The expected value is calculated in a process of summing up all possible outcomes which are then multiplied by their probabilities. The formula below illustrates this process:

\[ EV(X) = \sum_{i=1}^{n} [p_i \times v_i] \]

where: \( EV(X) \) is expected value of a random variable of a game or a bet \( X \), taking values \( v_i \), and \( p_i \) probabilities of getting the result.\(^{10}\)

In accordance with the expected value maximization principle one should decide to take the option with the highest expected value, namely the one where the sum of values multiplied by their probabilities is the highest. It is also known that human minds unconsciously lean towards estimated data whereupon the initial estimation anchors and restricts the viewpoint.\(^{11}\)

Expected utility theory proposed by Bernoulli is alternative to the one mentioned above. Bernoulli explained people's behavior referring to the differentiation between subjective and objective outcome value and he assumed that people tend to transform the explicitly given values into subjective values - utilities. Generally, people tend to distort the facts to bend them to their beliefs. In addition, they often overlook one cause of their behavior when another is more distinct and visible.\(^{12}\) The formula for the expected utility is as follows:

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12 Ibidem, p. 108.
\[ EU(X) = \sum_{i=1}^{n} [p_i \times u(x_i)] \] 

where: \( EU(X) \) is expected utility, \( u(x_i) \) is utility \( i \) of this outcome, and \( p_i \) is probability of this outcome.

Concluding, one can say that in their decisions the decision-makers should be driven by maximum expected utility where the sum of product probability of obtaining the outcome and utility connected with it is the highest.

The prospect theory presents a different approach than the theories presented above and it will be discussed in the next part of this paper.

2. The prospect theory

The expected utility theory assumes that if a decision-maker has a choice between gain \( x \) that is certain and the risky option of expected value \( y \), he should choose a certain gain because its expected utility is higher than that of the risky option. Hence the conclusion, that people should avoid risk at any price because when a person succeeds he/she credits the authorship to himself/herself whereas, in case of failure he/she assumes that its cause is rooted in external factors\(^{15}\). In truth, however, these models are often more or less diverged from reality, thus, many economists and psychologists of the 20\(^{th}\) century tried to answer the question whether people do avoid risk. The most renowned response was proposed in 1997 by D. Kahneman and A. Tversky in their prospect theory\(^{16}\).

Before they established the prospect theory, D. Kahneman and A. Tversky carried out numerous experiments – Table 1 presents the results of one of them.

In the table presented above in the first experiment the sampled under investigation were asked to choose one of the options in two situations. In the first situation they had a choice between a certain gain of 3000 PLN and a higher (by 1000 PLN) gain, which was possible to be achieved with 0.8 probability (80% of the samples under investigation chose certain gain due to the loss risk aversion). In the second situation one was sure to lose 3000 PLN or to lose 4000 PLN with 0.2 probability (in this case reverse preferences were observed – 92% of the samples under investigation had chosen uncertain loss).

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16 J. Sokołowska, Psychologia…, op. cit., p. 111.
On this basis scientists D. Kahnemann and A. Tversky defined two effects that take place in a situation when there is a choice between more or less certain gains and losses. The certainty effect denominates the tendency to prefer a lower but certain gain to a higher but risky one. The reflection effect, in turn, denominates the tendency to prefer a higher and uncertain loss against a certain but lower one\(^{17}\). It may be related to the law of effect which states that if a certain behavior produces a satisfying effect in a particular situation, it is more likely to occur again in a similar situation, and responses that produce a discomforting effect will become less likely to occur again in a similar situation\(^{18}\).

Corresponding effects were obtained in the experiment in which there were two lotteries to take part in – the results of the experiment are shown in Table 2.

In the experiment presented in Table 2 the sample group had to deal with two situations with two possible choices. In the first situation they had a choice between a quite certain gain (0.9 probability) of 3000 PLN and a 6000 PLN gain of 0.45 probability – a vast majority, namely 86% preferred a rather certain but lower gain. In the second situation the sample group had a chance to bid the option where they could lose 3000 PLN with 0.9 probability or the option where the possible loss was higher as it accounted for 6000 PLN but its probability dropped to 0.45. Again, facing the possible loss the samples under investigation had shown a strong aversion to certain loss

\(^{17}\) Ibidem, p. 113.

\(^{18}\) A. Trzciniecka-Green (edit.), *Psychologia. Podręcznik dla studentów kierunków medycznych*, UNIVERSITAS, Kraków 2006, p. 44.
and as much as 92% preferred to risk the loss of a bigger sum of money rather than to decide to accept a certain payout of 3000 PLN.

Table 2. Preferences of the sample group towards uncertain gain and loss in a lottery in D. Kahnemann and A. Tversky's experiment

<table>
<thead>
<tr>
<th>Situation 1</th>
<th>Option 1</th>
<th>Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain of 3000 PLN with 0.9 probability or no gain with 0.1 probability</td>
<td>Gain of 6000 PLN with 0.45 probability or no gain with 0.55 probability</td>
<td>86% of samples under investigation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Situation 2</th>
<th>Option 1</th>
<th>Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss of 3000 PLN with 0.9 probability or no loss with 0.1 probability</td>
<td>Loss 6000 PLN with 0.45 probability or no loss with 0.55 probability</td>
<td>8% of samples under investigation</td>
</tr>
</tbody>
</table>


Considering the results of the presented experiments it must be concluded, after D. Kahnemann and A. Tversky, that people in general avoid risk in possible gain situations whereas they are more ready to take it when there is a probability of a loss.

3. The standard utility function and the prospect theory

Conclusions presented in point 2 of the paper lead to the conclusion that a standard utility function presented in figure 1 does not clearly show preferences of average decision-makers as the function course should be different for gains and for losses.

Figure 1. Standard utility function

![U(x)](attachment:utility_function.png)

Source: Author’s own study based on: A. Solek, Optymalne decyzje. Ekonomia menedżerska w zadaniach, Wydawnictwo Uniwersytetu Ekonomicznego, in Krakow, Kraków 2008, p. 90; where $U(x)$ – utility e.g. of one's property, $x$ – one's property.

During the research conducted over the course of the utility function by the fathers of the prospect theory it turned out that in case of gains the course is in accordance with the classical
theory (e.g. while the money grows steadily, the utility can be characterized by a decreasing growth). Whereas, when losses are considered, the utility function takes a different shape. D. Kahneman and A. Tversky claimed that the utility functions in case of gains and losses can be represented by the power function. However, its parameters will be different in the case of negative and positive results. The utility of gains is represented by the following formula:

\[ u(x^+) = x^\alpha, \]

(3)

where: \( u \) is subjective value of the result, \( x^+ \) is the positive result and \( \alpha \) is an exponent of a power function for gains (equal to 0.88)\(^{19}\). The utility of losses is represented by the following formula:

\[ u(x^-) = -\lambda(-x)^\beta, \]

(4)

where: \( x^- \) is a negative result, \( \lambda \) is a constant of a power function describing losses (equal to 2.25) and \( \beta \) is an exponent of a power function for losses (equal to 0.88)\(^{20}\).

The utility function described above, also known as S-shaped curve is graphically represented by figure 2. The function consists of 2 segments: a convex segment which represents risk aversion in case of gains (the so called positive payout) and a concave segment which represents risk preferences in situations characterized by loss probability (the so called negative payout).

**Figure 2. Utility function in the prospect theory**

![Utility function curve](image)


Figure 2 presents the curve representing the utility function. The segment referring to losses is steeper which results from the tendency to reject fair lotteries (i.e. the lotteries where the chances to lose or to win identical sums are equal). Moreover, together with the rise of a bid the rejection frequency of such type of games rises. It means that, from the psychological point of view, people feel loss more strongly than gain. An interesting approach to losses is presented by T. Tyszka. He says that if we buy, let us say a cinema ticket and we lose it, then when we buy another one we feel

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\(^{19}\) J. Sokółowska, *Psychologia…*, op. cit., p. 115.

\(^{20}\) Ibidem.
as if we paid twice for it. However, when people lose the money intended to be spent on the ticket and spend different money to purchase it, the feeling of paying twice does not occur.\(^{21}\)

Another quality of the utility function in the prospect theory, which is visible in the part of the function graph referring to gains, is as follows: smaller but more frequent payouts are relatively more utile for decision-makers than more occasional but higher payouts. Hence, the conclusion: division of a bigger gain into few smaller payouts can lead to greater satisfaction than one bigger payout (figure 3).

**Figure 3. Dividing gains in the prospect theory**

![Utility Function Graph](image)


In the presented example two people who received lottery tickets drew $50 and $25 or $75, respectively. Person 1 should be more satisfied as upon consolidating the $50 and $25 winnings total utility is higher than the utility of a single $75 winning of a person 2. In that case we can observe a rule that people are more satisfied with two smaller gains than with one bigger gain. This fact is often used when rewarding employees.

When we observe the concave part of the utility graph in the prospect theory we can see a different regularity, this time referring to losses. It is presented in figure 4. In the graph shown above we can see that in case of losses bigger payouts are relatively less utile than smaller payouts. It means that, unlike in case of gains, people have stronger emotions about one bigger loss than few smaller losses.

4. Phases of the decision-making process in the prospect theory

D. Kahnemann and A. Tversky assumed that decision-making under risk distinguishes two phases\(^{22}\):

- the preliminary analysis of the problem in which the decision-maker analyses the offered prospects, estimates the consequences of every one of them and assigns appropriate probabilities to them. Framing, a vital element here, means fixing a reference point which differentiates between potential gains and losses;
- the evaluation of the options and choosing one of them, in other words, maximizing subjective expected value in place of expected utility.

Other division in the prospect theory distinguishes the editing phase and evaluation phases.

The editing phase is based on analysis – i.e. a mental processing of the decision-making situation, namely the following mechanisms can be involved in the process\(^ {23}\):

- \textit{coding} – refers to people's tendency to categorize outcomes in terms of gains and losses.
  As a reference point we can take decision-maker's holding or the state that is the outcome of the decision-maker's expectations (e.g. a worker who asked for a 200 PLN pay rise and got 100 PLN can consider it a loss);
- \textit{combination} – refers to consolidating the probabilities of identical outcomes or neglecting them when they are insignificant (e.g. a lottery with a possible results of \(v_1=200\) PLN, \(p_1=0.25\); \(v_2=200\) PLN, \(p_2=0.25\)) can add up to one result \((v=200\) PLN, \(p=0.5\));
- \textit{segregation} – refers to separating uncertain outcome (with a certain risk level) from a

\(^{22}\) P. Zielonka, \textit{Bezwarzialne aspekty inwestowania na rynku papierów wartościowych}, Wydawnictwa Fachowe CeDeWu.pl Warszawa 2006, p. 76.

certain one (e.g. a lottery with probable results \( v_1=400 \text{ PLN}, p_1=0.8; v_2=200 \text{ PLN}, p_2=0.2 \)) can be replaced with a certain 200 PLN and a lottery \( v=400 \text{ PLN}, p=0.8 \);

- **cancellation** – refers to the comparison of the elements which differ in particular alternatives (discounting common features in the analysis).

In the second stage of the prospect theory – the evaluation stage – two elements can be distinguished: the estimation of value and the estimation of chances. The estimation of value is described by the value function that corresponds with the expected utility function defined, in turn, upon changes referring to the decision-makers’ resources

Depending on the reference point (tracing back to the editing stage) it influences the way one can perceive the outcome as either a gain or loss. The perception of gains and losses has already been discussed. The second element of the estimation stage – the estimation of chances - takes place when arithmetic means are ascribed to particular outcomes. These means can constitute, in a way, the transformations of probabilities – Figure 5 presents the decision weighting function.

**Figure 5. Weighting function in the prospect theory**

![Decision weighting function](image)


While describing the weighting function presented in figure 5 one needs to pay attention to 4 fundamental properties:

- the growth within the range \([0;1]\), however in \((0)=0\) and in \((1)=1\);
- **subcertainty** – the means of opposite events do not add up to 1 as they do in the case of typical probabilities, however, often \(w(p)+w(1-p)<1\);
- a significant influence of random events on choices (greater than could be expected from their low probabilities), e.g. resignation from purchasing an insurance because of the low risk of damage and, on the other hand, taking part in lotteries where the chance of winning is really insignificant;

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24 Ibidem.
• perceiving the events of a very low probability as improbable and ignoring them, and on the other hand perceiving very probable events as absolutely certain.

5. Extension of the prospect theory

Thanks to the experiments D. Kahneman and A. Tversky extended the prospect theory and created the cumulated prospect theory. It consists of 4-part scheme of people's attitude towards risk with the following assumptions:

- people usually tend to avoid risk in favor of gains that are moderate in probability or extremely probable \((p \geq 0.5)\);
- people tend to risk in the case of moderate losses or highly probable losses \((p \geq 0.5)\);
- people tend to risk in the case of low probability gains \((p \leq 0.1)\);
- people tend to avoid risk when losses are of a very low probability \((p \leq 0.1)\).

The prospect theory and investigations that served to prove its assumptions have been widely implemented and are still used as a basis for deliberations over decision-makers' behaviors when they have to make decisions in risky situations. D. Kahnemann and A. Tversky's achievements have inspired i.a. R. Thaler and his mental accounting.

6. Author’s own research

The aim of the research was to compare the attitude of WSZiB (The School of Banking and Management) students to decision making under risk which would result either in a gain or loss. 166 people - representatives of all faculties - took part in the research, where 67% were women and 33% were men. The first part of the research concerned the self-estimation of the ability and frequency of making apt decisions under risk. The results of the research are presented in figures 6 and 7.

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28 Students up to 26 years of age accounted for 76%, from 27 to 35 years old accounted for 13%, above 36 years old - 11% of research group.
The vast majority of samples under investigation only occasionally make risky decisions (44%) and, interestingly enough, there is a small percentage of people who claim that they never make decisions of that kind. Moreover, people in general estimate quite highly their ability to make decisions (49%) and on average level (42%), which proves a high self esteem.

Another part of the research concerned specific choices which referred to gains with a certain level of probability. The results are presented in figures 8 and 9.

![Figure 8. The choice between two investment trusts](source: Author’s own study)

![Figure 9. The choice between investing in two types of shares](source: Author’s own study)

While making choices between a lower and 100% certain gain and a higher but 80% probable gain the samples under investigation did not see any difference. They opted 50-50 for each possibility and it does not confirm the prospect theory which says that the majority of people would rather choose lower but 100% certain gains. The prospect theory, however, found its justification in the choice of shares where 77% of samples under investigation chose a lower but more certain gain.

Another research concerned the students’ attitude towards losses – generally avoided by people. The results are presented in a graphic form in figures 10 and 11.

On the basis of the research it can be concluded that, in accordance with the prospect theory, the majority of students (81%) showed aversion to 100% certain losses. It turned out that the samples under investigation prefer to risk a higher loss, which can occur with a lower probability, rather than to risk a lower but 100% certain loss. Quite different preferences can be observed among people in the case of losses where a higher amount of money and lower probability is concerned as well as a lower sum of money and higher probability. The samples under investigation did not actually see any difference between the available options.
Figure 10. The choice between two portfolios

- 81%: certain loss 2000PLN
- 15%: loss 3000PLN (p=0.2); no loss or gain (p=0.8)

Figure 11. The choice between two types of shares

- 50%: loss 10000PLN (p=0.9); no loss or gain (p=0.1)
- 50%: loss 40000PLN (p=0.45); no loss or gain (p=0.55)

Source: Author’s own study

Satisfaction with the choices made by the decision-makers was also analyzed from the mental accounting point of view – the results are presented in figures 12 and 13.

Figure 12. The choice of a message causing greater satisfaction – combining a higher gain with lower loss

- 32%: 7000PLN winnings in one lottery and 5000PLN loss in other
- 27%: 2000PLN winnings in a lottery
- 41%: in both cases I would be equally satisfied

Source: Author’s own study.

The acquired results did not confirm the prospect theory, according to which people are satisfied with a higher gain related with a lower loss more than with a lower gain alone. The majority of samples under investigation chose lower gain with no losses, despite the fact that choosing a higher gain and suffering a loss they had the same amount of money in the end.

Figure 13. The choice of a message causing greater satisfaction – separating gains

- 63%: 2000PLN winnings in one lottery and 3000PLN loss in other
- 17%: 5000PLN winnings in a lottery
- 20%: in both cases I would be equally satisfied

Source: Author’s own study.

Another question that the samples under investigation had to answer partially confirmed the assumptions of the prospect theory – the majority of samples under investigation (63%) chose two lower gains instead of one smaller gain.

In the last part of the research the level of concern in the case of loss was investigated. It was also based on mental accounting – the results are presented in figures 14 and 15.
Figure 14. The choice of a message causing greater concern – combining losses

Source: Author’s own study.

In the case of losses the assumptions of the prospect theory that say that people perceive one high loss more intensely than two lower losses were not confirmed as it turned out that the bigger percentage of samples under investigation (21%) would prefer to lose smaller amount of money twice rather than to lose a bigger amount of money once (19%). There was a slight difference in choices, hence it is difficult to talk about any regularities here.

Figure 15. The choice of a message causing greater concern – separating a higher loss from lower gain

Source: Author’s own study.

From figure 15 it can be concluded that people are less disappointed by a loss that occurs when we subtract a lower gain (17%) from a higher loss than they would be just in the case of a loss (42%). This is not in line with the mental accounting, which assumes the separation of a higher loss from a lower gain.

Conclusion

The prospect theory in behavioral economics which, to a great extent, is built on empirical investigations, is useful when analyzing the decisions made by decision-makers. Its main assumption is that people generally show risk aversion in case of a gain and are more prone to it in case of losses (in order to avoid them). That sound and logical assumption means that the prospect theory can be used when analyzing people's choices in every day life.

The paper presented the results of the investigations which did not confirm some of the prospect theory assumptions. In the case of gains, the samples under investigation did not show a high level of risk aversion and the majority of them did not apply the rules of mental accounting. Only when analyzing the results of the question referring to 100% lower loss and uncertain higher loss can one see that the samples under investigation preferred to take risk hoping to avoid loss of
any sum of money. The results of the research, however, do not attest that the prospect theory is not right, as the research was carried out among the students of just one college (The School of Banking and Management) and the results can only be interpreted within the frames of this very institution.

Bibliography


Summary

The paper aims at presenting the prospect theory - one of the theories in behavioral economics for decision making by decision-makers. The theory created by D. Kahnemann and A. Tversky shows how people make decisions under risk and what their attitude to gains and losses is. For the purpose of this paper the research was carried out, whose results are presented and compared with the assumptions of the prospect theory.